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Piezomagnetic fields arising from the propagation of teleseismic waves in magnetized crust with finite conductivity

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SUMMARY

To determine whether the piezomagnetic effect is a plausible mechanism in explaining variations in the magnetic field that occur synchronously with the propagation of teleseismic waves, a set of solutions are derived for the electromagnetic field. The situation is considered in which the Earth's conductivity has a stratified structure and seismic waves are expressed as a plane wave. The piezomagnetic field in this situation is expressed by an analytically closed form. Using the obtained solution, quantitative aspects of the piezomagnetic field that accompanies seismic Rayleigh waves are discussed. It is shown that the finite conductivity of the Earth's crust sometimes acts as an enhancer of the magnitude of the piezomagnetic field. However, the expected piezomagnetic field is substantially small. Even in the case that the initial magnetization around the observation site is as large as 5 A m^{-1}, the expected amplitudes in the piezomagnetic field are at most 0.1 nT. This result means that the piezomagnetic effect is not a reasonable mechanism to sufficiently explain variations in magnetic fields that occur synchronously with ground motions, if the initial magnetization is horizontally uniform.

Key words: Electromagnetic theory; Magnetic and electrical properties; Earthquake ground motions; Wave propagation.

1 INTRODUCTION

Variations in electromagnetic (EM) fields following large earthquakes are frequently reported (e.g. Eleman 1965; Honkura *et al*. 2000; Iyemori *et al*. 1996, 2005; Taira *et al*. 2009). Such EM variations occur naturally, as theoretical considerations predict that changes in stress and displacement of the crust cause EM variations of various types. The candidate mechanisms for such variations include the electrokinetic effect (e.g. Mizutani *et al*. 1976; Ishido & Mizutani 1981), the EM induction effect due to ground motions (e.g. Honkura *et al*. 2002) and piezoelectricity (e.g. Ikeda 1990). The problem to be addressed is which mechanism(s) is dominant in the generation of EM variations. The electrokinetic effect is usually considered a major effect that converts seismic waves to variations in EM fields (e.g. Bordes *et al*. 2008). A recent study proposed that the motions of ions caused by the Lorentz force in the geomagnetic field are also important (Honkura *et al*. 2009). The contribution of the piezoelectric field has also been quantitatively discussed (e.g. Ogawa & Utada 2000a, 2000b). An understanding of the basic properties of signals arising from each mechanism is essential from both theoretical and applied perspectives.

One of the candidate mechanisms that generate coseismic variations in the magnetic field is the piezomagnetic effect. The piezomagnetic effect generates changes in magnetization in the Earth's crust under the application of mechanical stress (e.g. Nagata 1970a; Stacey & Johnston 1972). Changes in the magnetic field due to the piezomagnetic effect, which are referred to as the piezomagnetic field, have been calculated for a variety of elastic models (e.g. Sasai 1991; Utsugi *et al*. 2000; Okubo & Oshiman 2004). In these earlier studies, only permanent displacements of elastic materials were considered. To investigate variations in the magnetic field during the propagation of seismic waves, the calculation should be extended in such a way that time-varying stress fields are properly treated. In the Stokes' solution of the theory of elasticity, permanent displacements are represented by near-field terms, whereas seismic waves are mainly represented by far-field terms (e.g. Aki & Richards 2002). Consideration of elastic waves corresponding to the far-field terms is essential when considering the piezomagnetic field observed at sites located far (i.e. several tens of kilometres) from the seismic source.

The fundamental questions related to EM signals associated with seismic waves via the piezomagnetic effect are as follows: (i) Which factors control the expected amplitude of the signal? and (ii) Are the signals detectable? Of course, the answers to several aspects of the first question are obvious. For example, large seismic waves generate large piezomagnetic signals because of the linearity of the piezomagnetic effect. However, other aspects of this question remain poorly understood, including how the Earth's conductivity affects the magnitude of the resultant piezomagnetic field.

In this paper, an analytical solution is derived of the EM field that arises accompanying teleseismic waves, as generated by the piezomagnetic effect. Quantitative properties of the piezomagnetic signals are deduced from the derived solution. The reminder of this paper is organised as follows: In Section 2, the governing equations and assumptions are described for the problem of interest. In Section 3, an analytical solution to the problem is derived. In Section 4, the solution is applied to a two-layer Earth model to gain a quantitative insight into the piezomagnetic field. Finally, the main conclusions are presented in Section 5.

2 DEFINITION OF THE PROBLEM

2.1 Governing equations in the time domain

The situation considered in this study is summarized in Fig. 1. A layered structure in conductivity is assumed. Initial magnetization in the crust is assumed to be uniform above the depth of the Curie point. Seismic plane waves are considered as sources of the piezomagnetic effect. Variations in stress generate variations in magnetization via the mechanism of the piezomagnetic effect, which in turn causes variations in the EM field. The governing equations of the above situation are the Maxwell equations of the EM field and the constitutive law of the piezomagnetic effect. For simplicity, the electrical permeability, rigidity and Poisson's ratio are assumed to be uniform. The relevant notation is summarized in Table 1.

2.1.1 The electromagnetic field arising from time-varying magnetizations

Fluctuations in the electric and magnetic fields ($\Delta\mathbf{E}$ and $\Delta\mathbf{B}$, respectively) arising from propagating seismic waves are investigated. In the case of no electrical charge in the material, $\Delta\mathbf{E}$ and $\Delta\mathbf{B}$ satisfy the Maxwell equations in materials:

$$\nabla \cdot [\varepsilon_0 \Delta\mathbf{E}(\mathbf{x}, t)] = 0, \tag{1}$$

$$\nabla \cdot \Delta\mathbf{B}(\mathbf{x}, t) = 0, \tag{2}$$

$$\nabla \times \Delta\mathbf{E}(\mathbf{x}, t) + \frac{\partial}{\partial t}\Delta\mathbf{B}(\mathbf{x}, t) = \mathbf{0} \tag{3}$$

and

$$\nabla \times \left[\frac{1}{\mu_0^m}\Delta\mathbf{B}(\mathbf{x}, t) - \Delta\mathbf{M}(\mathbf{x}, t)\right] = \mathbf{j}(\mathbf{x}, t) + \varepsilon_0 \frac{\partial}{\partial t}\Delta\mathbf{E}(\mathbf{x}, t), \tag{4}$$

where $\mathbf{j}$ is the electric current density; ε_0 and μ_0^m are the electric and magnetic permeability in a vacuum, respectively; and $\Delta\mathbf{M}$ represents temporal variations in magnetization. Ohm's law is also assumed for $\Delta\mathbf{E}$ and $\mathbf{j}$; that is,

$$\mathbf{j}(\mathbf{x}, t) = \sigma \Delta\mathbf{E}(\mathbf{x}, t), \tag{5}$$

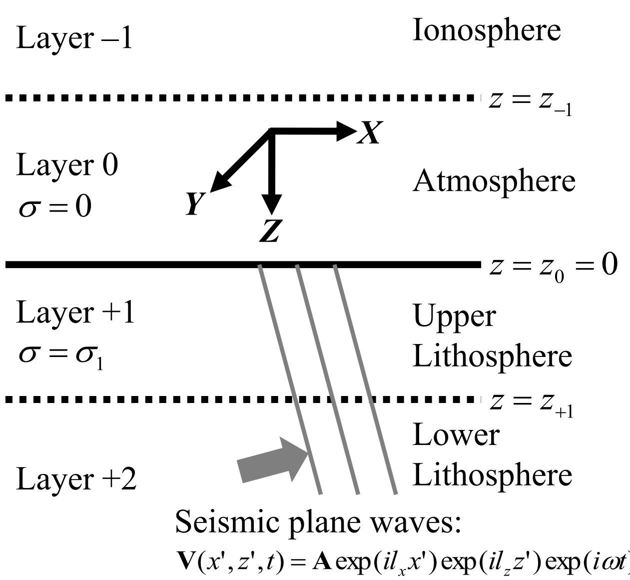

Figure 1. Configuration of the problem. The X-axis indicates the propagation direction of seismic waves in the horizontal plane; the Z-axis is vertically downwards and the Y-axis is normal to the XZ plane. The large arrow and oblique lines in grey represent an example of the propagation direction and wave fronts of seismic body waves, respectively. The two-layer model includes Layers 0 and 1 for consideration.

Table 1. Notation employed for variables and parameters.

Notation		Unit
$\Delta\mathbf{B}$	Magnetic field due to the piezomagnetic effect	T
$\Delta\mathbf{B}_{\lvert x-x'\rvert<W}$, $\Delta\mathbf{B}_{\lvert x-x'\rvert\geq W}$	See eq. (19) and associated text	T
$\Delta\mathbf{E}$	Electric field due to the piezomagnetic effect	V m^{-1}
Π^{m}	Magnetic Hertz potential	Tm^2
γ_{ij}^{Π}	3-D fundamental solution of Π^{m}	Tm^2
Γ_{ij}^{Π}	2-D fundamental solution of Π^{m}	Tm^2
Γ_{ij}^{B}	2-D fundamental solution of $\mathbf{B}$ or $\Delta\mathbf{B}$	T
Γ_{ij}^{E}	2-D fundamental solution of $\mathbf{E}$ or $\Delta\mathbf{E}$	Vm^{-1}
σ	Electrical conductivity	S m^{-1}
ω	Frequency (of seismic waves and EM waves)	s^{-1}
$\mathbf{M}$	Initial magnetization	Am^{-1}
$\Delta\mathbf{M}$	Piezomagnetization	Am^{-1}
g	Integral variable in integral transforms	m^{-1}
u_n	Quantity defined in eq. (29)	m^{-1}
$U_n^{\parallel}$, $U_n^{\perp}$, $D_n^{\parallel}$, $D_n^{\perp}$	Functions of g and z'	(none)
$\mathbf{V}$	Displacement vector	m
x, y, z	Location of observation points	m
x', y', z'	Location of sources of the EM field	m
μ_0^m	Magnetic permeability in a vacuum $= 4\pi \times 10^{-7}$	(none)
μ^e	Rigidity	Pa
β	Stress sensitivity of the piezomagnetic effect	Pa^{-1}
H	Curie point depth	m
l_x, l_y, l_z	Wavenumbers in the x, y and z directions	m^{-1}
$\mathbf{A}$	Amplitude vector of seismic waves	m
A_R	Amplitude of Rayleigh waves	m

where σ represents electrical conductivity. The conductivity is assumed to be a scalar function of the depth (z). By substituting Ohm's law, eq. (4) is reduced to

$$\nabla \times \Delta\mathbf{B}(\mathbf{x}, t) - \mu_0^m \sigma \Delta\mathbf{E}(\mathbf{x}, t) + \varepsilon_0 \mu_0^m \frac{\partial}{\partial t} \Delta\mathbf{E}(\mathbf{x}, t) = \mu_0^m \nabla \times \Delta\mathbf{M}(\mathbf{x}, t). \tag{6}$$

The third term on the left-hand side of this equation represents the displacement current, which is traditionally ignored, yielding

$$\nabla \times \Delta\mathbf{B}(\mathbf{x}, t) - \mu_0^m \sigma \Delta\mathbf{E}(\mathbf{x}, t) = \mu_0^m \nabla \times \Delta\mathbf{M}(\mathbf{x}, t). \tag{7}$$

For a given $\Delta\mathbf{M}$, eqs. (1)–(3) and (7) uniquely determine $\Delta\mathbf{E}$ and $\Delta\mathbf{B}$.

2.1.2 Time-varying magnetization due to the piezomagnetic effect

Experimental and theoretical studies have shown that changes in magnetization due to the piezomagnetic effect are approximately proportional to the product of the applied stress and the initial magnetization when the amplitude of applied stress is the same order of magnitude as that in the Earth's crust (e.g. Stacey 1964; Ohnaka & Kinoshita 1968; Nagata 1970b; Stacey & Johnston 1972; Zlotnicki *et al*. 1981). The constitutive laws of the linear piezomagnetic effect are summarized as the following simple relation (Sasai 1991):

$$\Delta\mathbf{M} = \frac{3}{2}\beta\mathbf{T}\mathbf{M}, \tag{8}$$

$$T_{ij} = \tau_{ij} - \frac{1}{3}\delta_{ij}(\tau_{xx} + \tau_{yy} + \tau_{zz}),$$

where $\Delta\mathbf{M}$ is the change in magnetization, $\mathbf{M}$ is the initial magnetization, $\mathbf{T}$ is the deviatoric stress tensor, τ_{ij} is a component of the stress tensor, β is the stress sensitivity and δ_{ij} is Kronecker's Delta. Using Hooke's law in an elastic medium, the constitutive law (eq. 8) is rewritten in the form

$$\Delta M_i = \beta\mu^e \sum_{j=x,y,z} \left[-\delta_{ij} \sum_{k=x,y,z} \frac{\partial V_k}{\partial x_k} + \frac{3}{2}\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right) \right] M_j, \tag{9}$$

where μ^e is rigidity, V_i is the i-component of the displacement field, and x_x, x_y and x_z are the coordinates that describe the position.

Far-field seismic waves in the elastic half-space are considered as the origin of changes in magnetizations. Seismic waves in the far-field region are expressed by the superpositions of plane waves. Plane waves that arise from a single seismic event propagate in the same direction. In addition, it is now assumed that all the physical properties of the Earth are horizontally uniform. Therefore, the x- and y-axes can be chosen in such a way that the y component of the wavenumber vector is zero, without any loss of generality. Displacement fields for the seismic plane wave are thus expressed in the form of

$$\mathbf{V}(x, z, t) = \mathbf{A}\exp[i(l_x x + l_z z)]\exp(-i\omega t), \tag{10}$$

where $\mathbf{A}$ represents the amplitude vector, l_i $(i = x, y, z)$ is the wavenumber in the respective direction and ω is the angular frequency. Substituting this expression into the constitutive law of the piezomagnetic effect (eq. 9), we have

$$\Delta M_i(x, z, t) = C_i \exp(il_x x)\exp(il_z z)\exp(-i\omega t), \tag{11}$$

with

$$C_i = i\beta\mu^e \sum_{j=x,y,z}\left[-\delta_{ij}\sum_{k=x,y,z} l_k A_k + \frac{3}{2}(l_i A_j + l_j A_i)\right] M_j.$$

The expression of a seismic wave with eq. (10) is valid only if the wave has a single wavenumber and frequency. A more general form is

$$\mathbf{V}(x, z, t) = \int_{-\infty}^{\infty} \mathrm{d}\omega \sum_{\alpha} \mathbf{V}(x, z, t; \omega)^{\alpha}, \tag{12}$$

$$\mathbf{V}(x, z, t; \omega)^{\alpha} = \mathbf{A}(\omega)^{\alpha} \exp\{i[l(\omega)_x^{\alpha} x + l(\omega)_z^{\alpha} z\}\exp(-i\omega t),$$

where α denotes the index for each plane wave. Any displacement fields that satisfy $\mathbf{V}(t = -\infty) = \mathbf{V}(t = +\infty)$ are expressed in this form. An expression of Rayleigh waves in the form of eq. (12) is given in Appendix A. An expression of time-varying piezomagnetization corresponding to the expression of eq. (12) is given by

$$\Delta\mathbf{M}(x, z, t) = \int_{-\infty}^{+\infty} \mathrm{d}\omega \sum_{\alpha} \Delta\mathbf{M}(x, z, t; \omega)^{\alpha}, \tag{13}$$

$$\Delta\mathbf{M}(x, z, t; \omega)^{\alpha} = \mathbf{C}(\omega)^{\alpha} \exp\left[il(\omega)_x^{\alpha} x\right]\exp\left[il(\omega)_z^{\alpha} z\right]\exp(-i\omega t),$$

$$C(\omega)_i^{\alpha} = +i\beta\mu^e \sum_{j=x,y,z}\left\{-\delta_{ij}\sum_{k=x,y,z} l(\omega)_k^{\alpha} A(\omega)_k^{\alpha} + \frac{3}{2}\left[l(\omega)_i^{\alpha} A(\omega)_j^{\alpha} + l(\omega)_j^{\alpha} A(\omega)_i^{\alpha}\right]\right\} M_j.$$

Because of the linearity of the governing equations, the resultant piezomagnetic field is given in the form of

$$\Delta\mathbf{B}(x, z, t) = \int_{-\infty}^{\infty} \mathrm{d}\omega \sum_{\alpha} \Delta\mathbf{B}(x, z, t; \omega)^{\alpha}, \tag{14}$$

where $\Delta\mathbf{B}(t; \omega)^{\alpha}$ represents the piezomagnetic field arising from the time-varying magnetization $\Delta\mathbf{M}(t; \omega)^{\alpha}$ defined in eq. (13).

2.2 Governing equations in the frequency domain

Because the sources of EM variations are expressed in the form of eq. (13), it is reasonable to solve the Maxwell equations in the frequency domain. Now that only far-field terms of the displacement are considered, all physical quantities, including $\Delta\mathbf{M}$, $\Delta\mathbf{B}$ and $\Delta\mathbf{E}$, satisfy the condition of $f(-\infty) = f(+\infty)$, where f represents a physical quantity as a function of time t. This condition assures the existence of the Fourier transform of f concerning the time t, which is given by

$$f(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(t)\exp(+i\omega t)\,\mathrm{d}t. \tag{15}$$

This definition follows a sign convention, which is usually adopted in seismology (e.g. column 5.2 in Aki & Richards 2002). Below, the same symbol is assigned to functions in the frequency and time domains, although distinguished by the variable: t indicates the function is defined in the time domain, whereas ω indicates the function is defined in the frequency domain.

By taking the Fourier transforms of the Maxwell equations (eqs 1–4) and the constitutive law of the piezomagnetic effect (eq. 13), governing equations in the frequency domain are obtained as follows:

$$\nabla \times \Delta\mathbf{E}(\mathbf{x}, \omega) - i\omega\Delta\mathbf{B}(\mathbf{x}, \omega) = \mathbf{0}, \tag{16}$$

$$\nabla \times \Delta\mathbf{B}(\mathbf{x}, \omega) - \mu_0^m \sigma \Delta\mathbf{E}(\mathbf{x}, \omega) = \mu_0^m \nabla \times \Delta\mathbf{M}(\mathbf{x}, \omega) \tag{17}$$

and

$$\Delta\mathbf{M}(x, z, \omega) = \sum_{\alpha} \mathbf{C}(\omega)^{\alpha} \exp\left[il(\omega)_x^{\alpha} x\right]\exp\left[il(\omega)_z^{\alpha} z\right], \tag{18}$$

where $\mathbf{C}(\omega)^{\alpha}$ is a function of ω defined in eq. (13).

2.3 Approximating transient seismic waves by persistent waves

The procedure used to calculate exact values of $\Delta\mathbf{B}(t)$ (and $\Delta\mathbf{E}(t)$) is as follows: first, determine $\mathbf{V}(\omega)$ based on the entire time-series of seismograms $\mathbf{V}(t)$, then calculate $\Delta\mathbf{B}(\omega)$ corresponding to each $\Delta\mathbf{M}(\omega)$ by means of eqs (16)–(18), and finally calculate the inverse Fourier transform of $\Delta\mathbf{B}(\omega)$ to obtain $\Delta\mathbf{B}(t)$. Because ground motions due to seismic waves are transient phenomena, the spectrum $\mathbf{V}(\omega)$ has

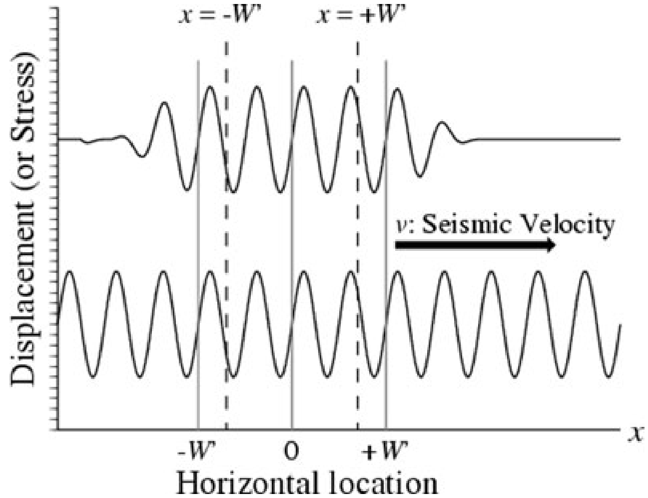

Figure 2. Example of (a) a transient seismic wave and (b) its approximation by a persistent wave at a specific time (i.e. $t = 0$; t represents time). The waveform of the persistent wave is the same throughout the range of $-W' < x' < +W'$, and has a spatial periodicity of $2W'$. Changes in magnetization at a distance greater than W make only a minor contribution to the magnetic field at the observation point, because of the rapid decay of the magnetic field with increasing distance from the source. Consequently, if $W < W'$, the piezomagnetic field calculated for a persistent wave provides a good approximation of the actual seismic wave for a limited period.

significant values throughout a relatively wide range of ω. Therefore, determination of the exact values of $\Delta\mathbf{B}(t)$ through the reverse Fourier transforms of $\Delta\mathbf{B}(\omega)$ is laborious.

Fortunately, an approximate estimation of $\Delta\mathbf{B}(t)$ can be made without such a laborious calculation, as follows: For an arbitrary distance W, $\Delta\mathbf{B}(t)$ at a location (x, y, z) is separated into two parts:

$$\Delta\mathbf{B}(t) = \Delta\mathbf{B}_{|x-x'|\le W}(t) + \Delta\mathbf{B}_{|x-x'|>W}(t), \tag{19}$$

where $\Delta\mathbf{B}_{|x-x'|\le W}$ and $\Delta\mathbf{B}_{|x-x'|>W}$ represent magnetic fields arising from magnetizations at (x', y', z') with $|x - x'| \le W$ and $|x - x'| > W$, respectively. The rapid decay of the magnetic field with increasing distance from the sources assures an inequality:

$$|(\Delta\mathbf{B}_{|x-x'|>W})_x| \le \frac{\mu_0^m}{\pi} |\Delta\mathbf{M}|_{\max} H \times W^{-1}, \tag{20}$$

where $|\Delta\mathbf{M}|_{\max}$ represents the maximum intensity of magnetization and H is the depth of the Curie point. The relevant proof is given in Appendix B. Seismic waves in a finite spatial range (i.e. $-W' < x' < +W'$) are regarded as a 'clipping' of a persistent wave with a periodicity W'. Inequality (20) indicates that the piezomagnetic field calculated for a persistent elastic wave provides a good approximation of the piezomagnetic field corresponding to the transient elastic wave, if the value of W is sufficiently large for $\Delta\mathbf{B}_{|x-x'|>W}$ to be ignored (Fig. 2).

In contrast to the case of transient waves, Fourier transforms of the persistent wave have significant values in a relatively narrow range of ω. Note that the spectra of seismograms are usually calculated in this way. Therefore, only $\Delta\mathbf{B}(\omega)$ from around the dominant frequency of seismic waves is required for estimating the piezomagnetic field, although this approach does not provide the exact time-series of EM variations. The same conclusion is also valid for $\Delta\mathbf{E}$ because variations in $\Delta\mathbf{E}$ are closely associated with those in $\Delta\mathbf{B}$.

3 INTEGRALS OF THE GOVERNING EQUATIONS

3.1 Fundamental solutions for magnetic Hertz vectors

The determination of EM fields arising from time-varying magnetic dipoles is more easily accomplished by introducing the magnetic Hertz vector $\mathbf{\Pi}^m$ than by directly considering the EM field (e.g. Stratton 1941). The magnetic Hertz vector is defined as a potential, from which the magnetic and electric fields are derived by

$$\Delta\mathbf{B}(\mathbf{x}, \omega) = \nabla \times (\nabla \times \mathbf{\Pi}^m(\mathbf{x}, \omega)), \tag{21}$$

$$\Delta\mathbf{E}(\mathbf{x}, \omega) = +i\omega\nabla \times \mathbf{\Pi}^m(\mathbf{x}, \omega). \tag{22}$$

Equations for the magnetic and electric fields (eqs 16 and 17, respectively) are reduced to three independent equations for components of the magnetic Hertz vector:

$$\left(\nabla^2 + i\omega\mu_0^m\sigma\right)\Pi_i^m(\mathbf{x}, \omega) = -\mu_0^m \Delta M_i(\mathbf{x}, \omega), \quad (i = x, y, z). \tag{23}$$

To find the solution of eq. (23), the following equation is first considered:

$$(\nabla^2 + i\omega\mu_0^m\sigma)\gamma_{ij}^{\Pi}(x, y, z, z', \omega) = -\mu_0^m\sigma\delta_{ij}\delta(x)\delta(y)\delta(z - z'), \tag{24}$$

where $\delta(\bullet)$ is Dirac's Delta function. The function γ_{ij} represents the i-component of the magnetic Hertz potential arising from the magnetic dipole moment in the j-direction located at $(0, 0, z')$. In this sense, γ_{ij}^{Π} is referred to as the point source solution. The solution of eq. (24) is given in the form of Hankel integrals, as follows:

$$\gamma_{xx}^{\Pi}(x,y,z,z',\omega) = \gamma_{yy}^{\Pi}(x,y,z,z',\omega) = \gamma_{zz}^{\Pi}(x,y,z,z',\omega)$$
$$= -\frac{\mu_0^m}{4\pi}\int_0^{+\infty}\left[\delta_{nn'}\frac{g}{u_{n'}}\exp(-u_{n'}|z-z'|) + U_n^{||}(g,z',\omega)\exp(+u_n z) + D_n^{||}(g,z',\omega)\exp(-u_n z)\right]\times J_0(gr)\,\mathrm{d}g, \tag{25}$$

$$\gamma_{zx}^{\Pi}(x,y,z,z',\omega) = -\frac{\mu_0^m}{4\pi}\frac{\partial}{\partial x}\int_0^{+\infty}\frac{1}{g}\left[U_n^{\perp}(g,z',\omega)\exp(+u_n z) + D_n^{\perp}(g,z',\omega)\exp(-u_n z)\right]J_0(gr)\,\mathrm{d}g, \tag{26}$$

$$\gamma_{zy}^{\Pi}(x,y,z,z',\omega) = -\frac{\mu_0^m}{4\pi}\frac{\partial}{\partial y}\int_0^{+\infty}\frac{1}{g}\left[U_n^{\perp}(g,z',\omega)\exp(+u_n z) + D_n^{\perp}(g,z',\omega)\exp(-u_n z)\right]J_0(gr)\,\mathrm{d}g, \tag{27}$$

and

$$\gamma_{yx}^{\Pi} = \gamma_{xy}^{\Pi} = \gamma_{xz}^{\Pi} = \gamma_{yz}^{\Pi} = 0, \tag{28}$$

where $r = \sqrt{x^2+y^2}$; n and n' represent the layers to which z and z' belong, respectively; u_n is a function of g and ω, defined by

$$u_n(g,\omega) = \sqrt{g^2 - i\omega\mu_0^m\sigma_n}; \tag{29}$$

and $U_n^{||}$, $U_n^{\perp}$, $D_n^{||}$ and $D_n^{\perp}$ are functions of g, z' and ω, which are determined by the boundary conditions of the EM field. Terms involving $U_n^{||}$ and $U_n^{\perp}$ represent variations propagating upward, and those involving $D_n^{||}$ and $D_n^{\perp}$ represent variations propagating downwards. Therefore, $U_n^{||}$ and $U_n^{\perp}$ in the bottom layer, and $D_n^{||}$ and $D_n^{\perp}$ in the top layer are equal to zero. The above solution is given by Stoyer (1977), although the sign of ω in u_n is modified to follow the sign convention employed in this study (eq. 15). Once γ_{ij}^{Π} are determined, Π_i^m for an arbitrary distribution of magnetization ($\Delta\mathbf{M}$) is calculated by

$$\Pi_i^m(x,y,z,\omega) = \sum_{j=x,y,z}\int_{-\infty}^{+\infty}\mathrm{d}x'\int_{-\infty}^{+\infty}\mathrm{d}y'\int_0^H\mathrm{d}z'\cdot\gamma_{ij}^{\Pi}(x-x',y-y',z,z',\omega)\Delta M_j(x',y',z',\omega). \tag{30}$$

For this reason, γ_{ij}^{Π} is also referred to as the fundamental solution of the Hertz vector in 3-D problems.

In the case that magnetization does not depend on the variable y', the integral for y' in eq. (30) is executed beforehand to yield

$$\Pi_i^m(x,z,\omega) = \sum_{j=x,y,z}\int_{-\infty}^{+\infty}\mathrm{d}x'\int_0^H\mathrm{d}z'\cdot\Gamma_{ij}^{\Pi}(x-x',z,z',\omega)\Delta M_j(x',z',\omega), \tag{31}$$

where

$$\Gamma_{ij}^{\Pi}(x,z,z',\omega) = \int_{-\infty}^{+\infty}\gamma_{ij}^{\Pi}(x,y-y',z,z',\omega)\,\mathrm{d}y'$$
$$= -\int_{-\infty}^{+\infty}\gamma_{ij}^{\Pi}(x,y,z,z',\omega)\,dy. \tag{32}$$

Substituting eqs (25)–(28) into eq. (32), and using a formula for the Bessel function derived in Appendix C, the following expressions are obtained:

$$\Gamma_{xx}^{\Pi}(x,z,z',\omega) = \Gamma_{yy}^{\Pi}(x,z,z',\omega) = \Gamma_{zz}^{\Pi}(x,z,z',\omega)$$
$$= +\frac{\mu_0^m}{2\pi}\int_0^{\infty}\frac{1}{g}\left[\frac{g}{u_n}\delta_{nn'}\exp(-u_n|z-z'|) + U_n^{||}(g,z',\omega)\exp(+u_n z) + D_n^{||}(g,z',\omega)\exp(-u_n z)\right]\cos(gx)\,\mathrm{d}g, \tag{33}$$

$$\Gamma_{zx}^{\Pi}(x,z,z',\omega) = -\frac{\mu_0^m}{2\pi}\int_0^{\infty}\frac{1}{g}\left[U_n^{\perp}(g,z',\omega)\exp(+u_n z) + D_n^{\perp}(g,z',\omega)\exp(-u_n z)\right]\sin(gx)\,\mathrm{d}g, \tag{34}$$

and

$$\Gamma_{yx}^{\Pi} = \Gamma_{xy}^{\Pi} = \Gamma_{zy}^{\Pi} = \Gamma_{xz}^{\Pi} = \Gamma_{yz}^{\Pi} = 0. \tag{35}$$

The function Γ_{ij}^{Π} represents the magnetic Hertz potential arising from a unit line source expressed by $\delta_{ij}\delta(x)\delta(z-z')$. For this reason, it is referred to as the line source solution. It is also referred to as the fundamental solution of the Hertz vector in 2-D problems, because of expression (35).

3.2 Fundamental solutions for electric and magnetic fields

2-D fundamental solutions of the magnetic field (Γ_{ij}^{B}) and the electric field (Γ_{ij}^{E}) are defined as vectors, in which the magnetic and electric fields arising from magnetizations are, respectively, calculated as

$$\Delta B_i(x,z,\omega) = \sum_{j=x,y,z}\int_{-\infty}^{+\infty}\mathrm{d}x'\int_0^H\mathrm{d}z'\cdot\Gamma_{ij}^{B}(x-x',z,z',\omega)\Delta M_j(x',z',\omega), \tag{36}$$

and

$$\Delta E_i(x,z,\omega) = \sum_{j=x,y,z} \int_{-\infty}^{+\infty} \mathrm{d}x' \int_0^H \mathrm{d}z' \cdot \Gamma_{ij}^E(x-x',z,z',\omega)\Delta M_j(x',z',\omega). \tag{37}$$

Explicit forms of Γ_{ij}^B and Γ_{ij}^E are obtained by substituting 2-D fundamental solutions of Hertz vectors (eqs 33–35) into the definition of Hertz vectors (eqs 21 and 22). Solutions corresponding to $j=x$ are given by

$$\Gamma_{xx}^B(x,z,z',\omega) = -\frac{\mu_0^m}{2\pi}\int_0^\infty \left[u_n\left(\frac{u_n}{g}U_n^{||}(g,z',\omega)+U_{n\backslash}^{\perp}(g,z',\omega)\right)\exp(+u_nz)+u_n\left(\frac{u_n}{g}D_n^{||}(g,z',\omega)-D_n^{\perp}(g,z',\omega)\right)\exp(-u_nz)\right.$$
$$\left.+\,\delta_{nn'}\frac{1}{u_n}\frac{\mathrm{d}^2}{\mathrm{d}z^2}\exp(-u_n|z-z'|)\right]\cos(gx)\,\mathrm{d}g, \tag{38}$$

$$\Gamma_{zx}^B(x,z,z',\omega) = -\frac{\mu_0^m}{2\pi}\int_0^\infty \left[g\left(\frac{u_n}{g}U_n^{||}(g,z',\omega)+U_n^{\perp}(g,z',\omega)\right)\exp(+u_nz)-g\left(\frac{u_n}{g}D_n^{||}(g,z',\omega)-D_n^{\perp}(g,z',\omega)\right)\exp(-u_nz)\right.$$
$$\left.+\,\delta_{nn'}\frac{g}{u_n}\frac{\mathrm{d}}{\mathrm{d}z}\exp(-u_n|z-z'|)\right]\sin(gx)\,\mathrm{d}g, \tag{39}$$

and

$$\Gamma_{yx}^E(x,z,z',\omega) = -i\omega\frac{\mu_0^m}{2\pi}\int_0^\infty \left[\left(\frac{u_n}{g}U_n^{||}(g,z',\omega)+U_n^{\perp}(g,z',\omega)\right)\exp(+u_nz)-\left(\frac{u_n}{g}D_n^{||}(g,z',\omega)-D_n^{\perp}(g,z',\omega)\right)\exp(-u_nz)\right.$$
$$\left.+\,\delta_{nn'}\frac{1}{u_n}\frac{\mathrm{d}}{\mathrm{d}z}\exp(-u_n|z-z'|)\right]\cos(gx)\,\mathrm{d}g. \tag{40}$$

Solutions corresponding to $j=z$ are given by

$$\Gamma_{xz}^B(x,z,z',\omega) = -\frac{\mu_0^m}{2\pi}\int_0^\infty \left[u_nU_n^{||}(g,z',\omega)\exp(+u_nz)-u_nD_n^{||}(g,z',\omega)\exp(-u_nz)+\delta_{nn'}\frac{g}{u_n}\frac{\mathrm{d}}{\mathrm{d}z}\exp(-u_n|z-z'|)\right]\sin(gx)\,\mathrm{d}g, \tag{41}$$

$$\Gamma_{zz}^B(x,z,z',\omega) = +\frac{\mu_0^m}{2\pi}\int_0^\infty \left[gU_n^{||}(g,z',\omega)\exp(+u_nz)+gD_n^{||}(g,z',\omega)\exp(-u_nz)+\delta_{nn'}\frac{g^2}{u_n}\exp(-u_n|z-z'|)\right]\cos(gx)\,\mathrm{d}g, \tag{42}$$

and

$$\Gamma_{yz}^E(x,z,z',\omega) = -i\omega\frac{\mu_0^m}{2\pi}\int_0^\infty \left[U_n^{||}(g,z',\omega)\exp(+u_nz)+D_n^{||}(g,z',\omega)\exp(-u_nz)+\delta_{nn'}\frac{g}{u_n}\exp(-u_n|z-z'|)\right]\sin(gx)\,\mathrm{d}g. \tag{43}$$

Solutions for other sets of i and j which do not appear in the above expressions are zero, which leads to $\Delta B_y = 0$ and $\Delta E_x = \Delta E_z = 0$. In addition, the contribution of ΔM_y disappears because Γ_{iy}^B and Γ_{iy}^E $(i = x, y, z)$ are equal to zero.

3.3 Example of a two-layer model

The simplest situation that approximates the actual Earth is a two-layer model, in which only the atmosphere (Layer 0) and the lithosphere with uniform conductivity (Layer 1) are considered. The conductivity in Layer 0 is assumed to be zero, meaning $u_0 = g$. It is also assumed that magnetization occurs only within Layer 1. The explicit forms of $U_0^{||}$, $U_0^{\perp}$, $D_1^{||}$ and $D_1^{\perp}$ are determined in such a way that they satisfy the boundary conditions (see Appendix D), which are given by

$$U_0^{||}(g,z',\omega) = \frac{2g}{g+u_1}\exp(-u_1z'), \tag{44}$$

$$U_0^{\perp}(g,z',\omega) = -2\frac{g-u_1}{g+u_1}\exp(-u_1z'), \tag{45}$$

$$D_1^{||}(g,z',\omega) = \frac{g(g-u_1)}{u_1(g+u_1)}\exp(-u_1z') \tag{46}$$

and

$$D_1^{\perp}(g,z',\omega) = -2\frac{g-u_1}{g+u_1}\exp(-u_1z'). \tag{47}$$

For practical purposes, it is enough to determine the field in Layer 0. Given that most geomagnetic data are acquired near the ground surface, it is only necessary to obtain the expression for Layer 0 to estimate the EM field at these points. Because of the continuity of the field components, solutions in Layer 0 provide a good approximation of the expected value of the EM field even if observations are conducted slightly beneath the ground surface. In addition, it is reasonable to neglect the ionosphere for the sake of simplicity in the calculations. In this problem, variations in magnetization are assumed to exist only beneath the ground (Layer 1), meaning that only negligible disturbance of the EM field is expected at the height of the ionosphere.

3.4 Convolutions of fundamental solutions and magnetizations

The ranges of the integrals in eqs (36) and (37) are $-\infty \le x' \le \infty$ and $0 \le z' \le H$, where H represents the Curie point depth, because magnetizations exist only in this volume. To calculate the integral for x', expression of Dirac's Delta function is referred to (e.g. Arfken & Weber 1995):

$$\delta(l_x - g) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp[i(l_x - g)x'] \, dx', \tag{48}$$

which leads to the following two formulae:

$$\int_{-\infty}^{+\infty} \cos[g(x - x')] \exp(il_x x') \, dx' = \pi\delta(l_x - g) \exp(il_x x) + \pi\delta(l_x + g) \exp(-il_x x), \tag{49}$$

and

$$\int_{-\infty}^{+\infty} \sin[g(x - x')] \exp(il_x x') \, dx' = -i\pi\delta(l_x - g) \exp(il_x x) + i\pi\delta(l_x + g) \exp(-il_x x). \tag{50}$$

In addition, Dirac's Delta function has the following fundamental property:

$$\int_{-\infty}^{+\infty} \delta(k - g) f(g) \, dg = f(k), \tag{51}$$

where f represents an arbitrary function. Combining eqs (49)–(51), the following two formulae are obtained:

$$\int_{-\infty}^{+\infty} dx' \int_{0}^{+\infty} dg \, f(g) \cos[g(x - x')] \exp(il_x x') = \pi f(l_x), \tag{52}$$

and

$$\int_{-\infty}^{+\infty} dx' \int_{0}^{+\infty} dg \, f(g) \sin[g(x - x')] \exp(il_x x') = -i\pi f(l_x). \tag{53}$$

Because the spatial distribution of magnetization due to the piezomagnetic effect has the form of eq. (18), the integrals in eqs (36) and (37) for variables x' and g are completed using formulae (52) and (53). Then, the integral for z' is elementary. Therefore, convolutions of eqs (36) and (37) are obtained to yield a set of analytical expressions.

In the case of a two-layer model, $U_n^{||}$, $U_n^{\perp}$, $D_n^{||}$ and $D_n^{\perp}$ $(n = 0, 1)$ are given by eqs (44)–(47), respectively. The explicit forms of the magnetic and electric fields due to the piezomagnetic effect are given by

$$\Delta B_x(x, z, \omega) = \mu_0^m \left(-\frac{l_x u_1}{l_x + u_1} C_x + i \frac{l_x^2}{l_x + u_1} C_z \right) \frac{1 - \exp[-(u_1 - il_z)H]}{u_1 - il_z} \exp(il_x x) \exp(+l_x z), \tag{54}$$

$$\Delta B_z(x, z, \omega) = \mu_0^m \left(i \frac{l_x u_1}{l_x + u_1} C_x + \frac{l_x^2}{l_x + u_1} C_z \right) \frac{1 - \exp[-(u_1 - il_z)H]}{u_1 - il_z} \exp(il_x x) \exp(+l_x z), \tag{55}$$

and

$$\Delta E_y(x, z, \omega) = i\omega\mu_0^m \left(-\frac{u_1}{u_0 + u_1} C_x + i \frac{l_x}{u_1 + u_0} C_z \right) \frac{1 - \exp[-(u_1 - il_z)H]}{u_1 - il_z} \exp(il_x x) \exp(+l_x z), \tag{56}$$

with $u_1 = u_1(g = l_x) = \sqrt{l_x^2 - i\omega\mu_0^m \sigma_1}$ and C_i defined in eq. (11).

By considering the Taylor expansion of exponentials in eqs (54) and (55), the order estimation of $\Delta\mathbf{B}$ is obtained as

$$|\Delta B_i| \approx \frac{\mu_0^m}{\pi} |\mathbf{m}|_{\max} H l_x \quad (i = x, z). \tag{57}$$

Compared with inequality (20), an approximation,

$$\Delta\mathbf{B} \cong \Delta\mathbf{B}_{|x-x'| \le W}, \tag{58}$$

is confirmed for W that satisfies $W \gg l_x^{-1}$. Because the spatial lengths of sequences of seismic waves (i.e. W' in Fig. 2) are sufficiently longer than the reciprocals of their wavenumber, it is possible to choose W such that $W < W'$ and $W \gg l_x^{-1}$. Therefore, the magnetic field calculated for a persistent elastic wave provides a good approximation of the actual magnetic field that arises from a transient seismic wave. A similar consideration is applied for the associated electric field.

4 NUMERICAL EXAMPLES AND DISCUSSION

Using the obtained expressions of the EM field due to the piezomagnetic effect, the amplitudes of magnetic field variations are calculated for various parameter values. According to an earlier report that variations in the magnetic field occur simultaneously with Rayleigh waves (Taira *et al*. 2009), only Rayleigh waves are considered as the cause of changes in magnetization. The numerical evaluation is performed for a two-layer model. The lower layer, representing the Earth's lithosphere, is assumed to be a Poisson solid, for which the seismic velocities of P waves, S waves and Rayleigh waves (v_P, v_S and v_R, respectively) satisfy $v_S/v_P = 0.5774$ and $v_R/v_S = 0.9194$ (e.g. Sheriff & Geldart 1995). The amplitude of the magnetic field ($|\Delta B_x|$) is calculated for combinations of parameters: periods T of $10 \le T \le 100$ s; conductivities of $\sigma_1 = 1$, 10^{-2} and 10^{-4} S; and Curie point depths $H = 5$ and 15 km. Other parameters employed in the calculation are fixed to the values listed

Table 2. Values of parameters adopted in the numerical test shown in Fig. 3.

β	μ^e	M_x	M_z	v_P	A_R
2×10^{-9} Pa^{-1}	$35 \times 10^{+9}$ Pa	5 A m^{-1}	5 A m^{-1}	1×10^{-2} m s^{-1}	0.01 m

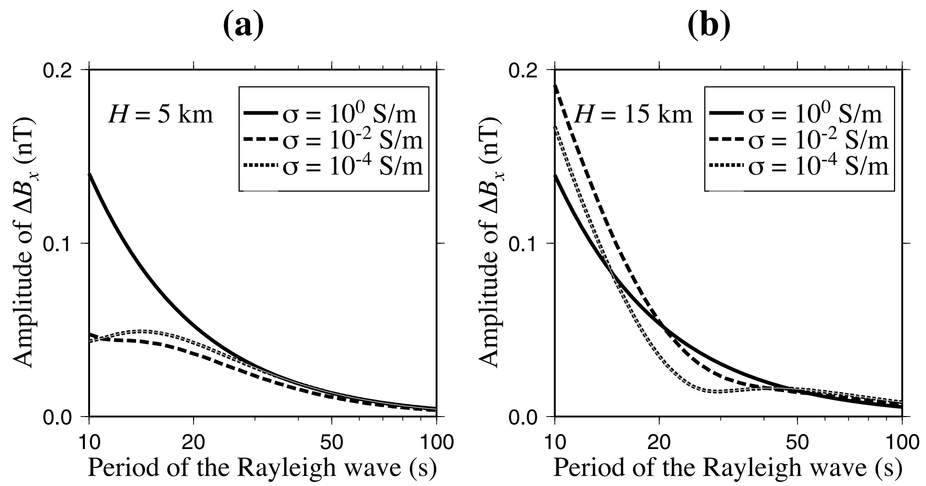

Figure 3. Amplitudes of the piezomagnetic field that accompany Rayleigh waves, plotted against the period of the wave (i.e. $T = 2\pi/\omega = 2\pi/(L_R v_R)$; L_R: wavenumber). Values are calculated using eq. (54) for Curie point depths of (a) 5 km and (b) 15 km. The observation height (z) is set to 0 m (at the ground surface). Assumed parameters are listed in Table 2. In each panel, results for the cases of $\sigma = 1$, 10^{-2} and 10^{-4} S m^{-1} are shown by solid, dashed and dotted lines, respectively.

in Table 2. Since the assumed intensities of the initial magnetizations ($M_x = M_z = 5$ A m^{-1}) are rather large, the corresponding estimation based on these parameters should be regarded as an upper limit. Note that eqs (54) and (55) yield $\Delta B_z = -i\Delta B_x$, so that consideration of only ΔB_x is sufficient for the order estimation of variations in the magnetic field. Eqs (54) and (56) also yield the simple relation $\Delta E_y = -v_R \Delta B_x$. Therefore, the estimation of variations in the electric field (ΔE_y) is straightforward, although it is not shown below. The results of the estimation of ΔB_x are plotted in Fig. 3.

An obvious feature of the results is the dependency of amplitude on period. The amplitude of the signal tends to decrease with increasing period, although with some exceptions. This feature is easily interpreted, as follows. The amplitudes of variations in stress are proportional to the wavenumber. Therefore, the intensity of magnetization due to the piezomagnetic effect is proportional to the wavenumber. Since wavenumbers are inversely proportional to the period of oscillation, the amplitude of the piezomagnetic field is reasonably expected to decrease with increasing wavenumber.

The dependency of ΔB_x on electrical conductivity σ_1 is an important feature. Variations in the EM field are converted to electric currents via EM induction. The energy of upward-propagating EM fields should be smaller in a conductive medium than in a resistive medium. Therefore, conductive crust is generally expected to act as an attenuator of variations in the EM field. However, the data in Fig. 3 are inconsistent with this explanation. Indeed, the expected magnitude of variations in the magnetic field is relatively large when the crust is conductive ($\sigma_1 = 1$ S m^{-1}). This contradiction is interpreted as follows. Induced currents that arise from the downward-propagating EM fields generate secondary magnetic variations, half of which propagate upwards. In this sense, the conductive crust acts as a reflector. The variations observed on the ground may be enhanced by the existence of the reflector. The magnitude of observed variations is determined by summation of these two opposite effects. Similar results are expected to be obtained in a more complex model, including heterogeneities in conductivities and initial magnetizations. One implication of the above consideration is that finite conductivity of the Earth should be included in the model when attempting to make an accurate estimate of the piezomagnetic field that accompanies seismic waves.

The most important implication of the above results is the small magnitude of the expected piezomagnetic fields, regardless of the dependency of the resultant piezomagnetic field on each variable. The dominant period (reciprocal of frequency) of Rayleigh waves is larger than 20–30 s. For these frequencies, the magnitudes of variations due to the piezomagnetic effect are up to 0.1 nT, which is close to the limit of detectability. The occurrence of more complex structures in the conductivity may enhance the signal. For example, when a conductor is embedded in a deep part of the lithosphere, it reflects the variations, meaning that the observed signal is enhanced. However, such amplification will not significantly alter the observability of variations in the magnetic field. In addition, the estimations are obtained for relatively high values of magnetization (i.e. $M_x = M_z = 5$ A m^{-1}). Most of the Earth's crust has smaller magnetizations than this value, meaning that the expected variations due to the piezomagnetic effect would be smaller than those estimated in the present examples. In summary, the piezomagnetic effect is unlikely to be a major mechanism, which explains fluctuations in the magnetic field accompanying the propagation of Rayleigh waves in far-field regions.

For situations in which the piezomagnetic field appears with a detectable magnitude, the configuration is different from those assumed in this study. Examples of such differences include heterogeneities in the initial magnetization of the crust and the inclusion of near-field terms of the displacement field. Regarding heterogeneities in initial magnetization, earlier studies on the piezomagnetic field, based on elastostatics, demonstrated that the piezomagnetic field is effectively enhanced near the boundaries of regions of intense magnetization (e.g. Oshiman 1990; Utsugi 1999; Yamazaki 2009). Such an enhancement effect is expected to be significant when considering the piezomagnetic field corresponding to the propagation of seismic waves. In the case of inhomogeneous magnetization, the convolutions of fundamental solutions are not represented in the forms of eqs (52) and (53). Consequently, they should be calculated numerically. The treatment of near-field terms is more difficult, although important. When the near-field terms are considered, it is no longer valid to approximate the displacement fields by persistent phenomenon with a periodicity (Fig. 2). Consequently, the present approach, which relies on the Fourier transform, cannot be applied. A formulation is currently being developed that treats the near-field terms at the expense of ignoring the Earth's conductivity (M. Utsugi, private communication, 2010).

5 CONCLUSIONS

For the case in which the conductivity of the Earth has a layered structure and magnetization is uniform, the piezomagnetic fields generated by a stress field caused by teleseismic waves is expressed by an analytically closed form. Numerical examples, employing the resultant expressions, show that the finite conductivity of the Earth's crust may enhance the piezomagnetic field observed on the ground surface. However, the expected amplitude of the piezomagnetic field is relatively small. The amplitudes corresponding to Rayleigh waves are up to 0.1 nT, even in the case that magnetization of the ambient crust is as large as 5 A m^{-1}. Therefore, piezomagnetic fields in the far-field of seismic events are not expected to be observed, if the initial magnetization of the Earth's crust is uniform.

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APPENDIX A: EXPLICIT FORMS OF RAYLEIGH WAVES

For simplicity, a Rayleigh wave is considered with a single frequency, so that $\exp(-i\omega t)$ is omitted in the following expression. The displacement fields of Rayleigh waves are given by the real parts of the following expressions (e.g. Lay & Wallace 1995):

$$V_x(x', z') = i A_R \exp(i L_R x') \left[\exp(-v_R\eta_P L_R z') + \frac{1}{2}\left(\frac{v_R^2}{v_S^2} - 2\right)\exp(-v_R\eta_S L_R z')\right], \tag{A1}$$

$$V_z(x', z') = -A_R \exp(i L_R x') \left[v_R\eta_P \exp(-v_R\eta_P L_R z') + \frac{1}{2v_R\eta_S}\left(\frac{v_R^2}{v_S^2} - 2\right)\exp(-v_R\eta_S L_R z')\right], \tag{A2}$$

where L_R is the wavenumber of the Rayleigh wave;

$$\eta_P = \sqrt{\frac{1}{v_R^2} - \frac{1}{v_P^2}}; \tag{A3}$$

$$\eta_S = \sqrt{\frac{1}{v_R^2} - \frac{1}{v_S^2}}; \tag{A4}$$

A_R is the amplitude of the Rayleigh wave; and v_P, v_S and v_R represent the velocities of P waves, S waves and Rayleigh waves, respectively. Eqs (A1) and (A2) are rewritten to the same form as eq. (12), by taking

$$\mathbf{V}(x', z', t) = \mathbf{A}^P \exp\left[i\left(l_x^P x' + l_z^P z'\right)\right]\exp(i\omega t) + \mathbf{A}^S \exp\left[i\left(l_x^S x' + l_z^S z'\right)\right]\exp(i\omega t), \tag{A5}$$

where

$$\mathbf{A}^P = (i A_R,\quad 0,\quad -v_R\eta_P A_R),\quad \mathbf{l}^P = (L_R,\quad 0,\quad i v_R\eta_P L_R), \tag{A6}$$

and

$$\mathbf{A}^S = \left(\frac{1}{2} i A_R\left(\frac{v_R^2}{v_S^2} - 2\right),\quad 0,\quad -A_R\frac{1}{2v_R\eta_S}\left(\frac{v_R^2}{v_S^2} - 2\right)\right),\quad \mathbf{l}^S = (L_R,\quad 0,\quad i v_R\eta_S L_R). \tag{A7}$$

APPENDIX B: PROOF OF AN INEQUALITY

This appendix provides a proof of inequality (20). For simplicity, the location of the observation point is fixed at (0, 0, 0). First, consider the case of a resistive Earth (i.e. $\sigma = 0$) for which the x-component of the magnetic field $\Delta\mathbf{B}$ arising from the magnetization $\Delta\mathbf{M} = (\Delta M_x, \Delta M_y, \Delta M_z)$ is given by

$$(\Delta\mathbf{B}_{|x'|>W})_x = -\frac{\mu_0^m}{4\pi}\left\{\int_{-\infty}^{-W} dx' + \int_{+W}^{+\infty} dx'\right\}\int_{-\infty}^{+\infty} dy' \int_0^H dz' \left[\frac{x'\Delta M_x + y'\Delta M_y + z'\Delta M_z}{(x'^2 + y'^2 + z'^2)^{3/2}}\right]. \tag{B1}$$

When $\Delta\mathbf{M}$ is a function of x' and z', and does not depend on y', the integral for variable y' is calculated to yield

$$(\Delta\mathbf{B}_{|x'|>W})_x = \frac{\mu_0^m}{2\pi}\int_W^\infty dx' \int_0^H dz' \left[\Delta M_x \frac{x'^2 - z'^2}{(x'^2 + z'^2)^2} + \Delta M_z \frac{2x'z'}{(x'^2 + z'^2)^2}\right]. \tag{B2}$$

The integrand is evaluated with the help of the Cauchy–Schwarz inequality:

$$\left|\Delta M_x \frac{x'^2 - z'^2}{(x'^2 + z'^2)^2} + \Delta M_z \frac{2x'z'}{(x'^2 + z'^2)^2}\right| \le |\Delta\mathbf{M}|\frac{1}{x'^2 + z'^2}. \tag{B3}$$

Hence, the absolute value of $(\Delta\mathbf{B}_{|x'|>W})_x$ satisfies the inequality

$$\begin{aligned}\left|(\Delta\mathbf{B}_{|x'|>W})_x\right| &< \frac{\mu_0^m}{2\pi}|\Delta\mathbf{M}|_{\max}\left\{\int_{-\infty}^{-W} dx' + \int_W^\infty dx'\right\}\int_0^H dz' \frac{1}{x'^2 + z'^2} \\ &< \frac{\mu_0^m}{2\pi}|\Delta\mathbf{M}|_{\max}\left\{\int_{-\infty}^{-W} dx' + \int_W^\infty dx'\right\}\int_0^H dz' \frac{1}{x'^2} = \frac{\mu_0^m}{\pi}|\Delta\mathbf{M}|_{\max} H W^{-1},\end{aligned} \tag{B4}$$

where $|\Delta\mathbf{M}|_{\max}$ represents the maximum intensity of $\Delta\mathbf{M}(x', z')$ in the region of $W \le x' < \infty$ and $0 \le z' \le H$. This inequality is the same as inequality (20). In the case of finite conductivity, the rate of decrease in increasing x is expected to be high because of attenuation of the EM field. Therefore, inequality (B4) is also assured. In a conductive Earth (i.e. $\sigma_1 \neq 0$), the magnetic field is expressed in a different form to (B1). However, the decay of a magnetic field with increasing distance x should be faster than that in a resistive Earth. Therefore, the inequality (B4) is still satisfied.

APPENDIX C: AN INTEGRAL OF THE BESSEL FUNCTION

We start from Neuman's additional theorem (Watson 1995), which leads to

$$J_0(g\sqrt{x^2+y^2}) = J_0(gx)J_0(gy) + 2\sum_{m=1}^{\infty}(-1)^m J_{2m}(gx)J_{2m}(gy). \tag{C1}$$

By integrating the above equation over the whole range of y, the following relation is obtained:

$$\int_{-\infty}^{+\infty} J_0(g\sqrt{x^2+y^2})\,\mathrm{d}y = \frac{2}{g}\left[J_0(gx) + 2\sum_{m=1}^{\infty}(-1)^m J_{2m}(gx)\right]. \tag{C2}$$

The Bessel functions appear in a Laurent series of its generating function:

$$\exp\left[\frac{1}{2}\left(t - \frac{1}{t}\right)gx\right] = \sum_{n=-\infty}^{\infty} t^n J_n(gx). \tag{C3}$$

Using the relation $J_{-n}(gx) = (-1)^n J_n(gx)$, the above expression is rewritten as

$$\exp\left[\frac{1}{2}\left(t - \frac{1}{t}\right)gx\right] = J_0(gx) + \sum_{n=1}^{\infty}[t^n + (-1)^n t^{-n}]J_n(gx). \tag{C4}$$

Putting $t = +i$ in the above equation, and comparing the real part of both sides, the following series expansion is obtained:

$$\cos(gx) = J_0(gx) + 2\sum_{m=1}^{\infty}(-1)^m J_{2m}(gx). \tag{C5}$$

A comparison between (C2) and (C5) yields

$$\int_{-\infty}^{+\infty} J(g\sqrt{x^2+y^2})\,\mathrm{d}y = \frac{2}{g}\cos gx. \tag{C6}$$

APPENDIX D: DETERMINATION OF $U_n^{||}$, $U_n^{\perp}$, $D_n^{||}$ AND $D_n^{\perp}$ BASED ON THE BOUNDARY CONDITIONS OF THE EM FIELD

The boundary conditions of an EM field are the continuity of the tangentials **E** and **B**. These conditions are applied to 2-D fundamental solutions. The continuities of Γ_{xx}^B, Γ_{yx}^E, Γ_{xz}^B and Γ_{yz}^E (eqs 38, 40, 41 and 43, respectively) are imposed at the boundaries ($z = z_n$), yielding

$$\begin{aligned} &u_n\left(\frac{u_n}{g}U_n^{||} + U_n^{\perp}\right)\exp(+u_n z_n) + u_n\left(\frac{u_n}{g}D_n^{||} - D_n^{\perp}\right)\exp(-u_n z_n) + \delta_{nn'}\frac{1}{u_n}\frac{\mathrm{d}^2}{\mathrm{d}z^2}\exp(-u_n|z_n - z'|) \\ &\quad = u_{n+1}\left(\frac{u_{n+1}}{g}U_{n+1}^{||} + U_{n+1}^{\perp}\right)\exp(+u_{n+1}z_n) + u_{n+1}\left(\frac{u_{n+1}}{g}D_{n+1}^{||} - D_{n+1}^{\perp}\right)\exp(-u_{n+1}z_{n+1}) \\ &\qquad + \delta_{n+1,n'}\frac{1}{u_{n+1}}\frac{\mathrm{d}^2}{\mathrm{d}z^2}\exp(-u_{n+1}|z_n - z'|), \end{aligned} \tag{D1}$$

$$\begin{aligned} &\left(\frac{u_n}{g}U_n^{||} + U_n^{\perp}\right)\exp(+u_n z_n) - \left(\frac{u_n}{g}D_n^{||} - D_n^{\perp}\right)\exp(-u_n z_n) + \delta_{nn'}\frac{1}{u_n}\frac{\mathrm{d}}{\mathrm{d}z}\exp(-u_n|z_n - z'|) \\ &\quad = \left(\frac{u_{n+1}}{g}U_{n+1}^{||} + U_{n+1}^{\perp}\right)\exp(+u_{n+1}z_n) - \left(\frac{u_{n+1}}{g}D_{n+1}^{||} - D_{n+1}^{\perp}\right)\exp(-u_{n+1}z_n) + \delta_{n+1,n'}\frac{1}{u_{n+1}}\frac{\mathrm{d}}{\mathrm{d}z}\exp(-u_{n+1}|z_n - z'|), \end{aligned} \tag{D2}$$

$$\begin{aligned} &u_n U_n^{||}\exp(+u_n z_n) - u_n D_n^{||}\exp(-u_n z_n) + \delta_{nn'}\frac{g}{u_n}\frac{\mathrm{d}}{\mathrm{d}z}\exp(-u_n|z_n - z'|) \\ &\quad = u_{n+1}U_{n+1}^{||}\exp(+u_{n+1}z_n) - u_{n+1}D_{n+1}^{||}\exp(-u_{n+1}z_n) + \delta_{n+1,n'}\frac{g}{u_{n+1}}\frac{\mathrm{d}}{\mathrm{d}z}\exp(-u_{n+1}|z_n - z'|), \end{aligned} \tag{D3}$$

and

$$\begin{aligned} &U_n^{||}\exp(+u_n z_n) + D_n^{||}\exp(-u_n z_n) + \delta_{nn'}\frac{g}{u_n}\exp(-u_n|z_n - z'|) \\ &\quad = U_{n+1}^{||}\exp(+u_{n+1}z_n) + D_{n+1}^{||}\exp(-u_{n+1}z_n) + \delta_{n+1,n'}\frac{g}{u_{n+1}}\exp(-u_{n+1}|z_n - z'|), \end{aligned} \tag{D4}$$

respectively. Arguments regarding $U_n^{||}$, $U_n^{\perp}$, $D_n^{||}$ and $D_n^{\perp}$ are omitted for simplicity. Physical requirements impose an additional two constraints:

$$U_M^{||} = U_M^{\perp} = 0, \tag{D5}$$

and

$$D_N^{||} = D_N^{\perp} = 0, \tag{D6}$$

where M and N represent the bottom and top layers, respectively. Eqs (D1)–(D6) determine $U_n^{||}$, $U_n^{\perp}$, $D_n^{||}$ and $D_n^{\perp}$ for $M \leq n \leq N$. In the particular case of a two-layer model, in which $M = 0$, $N = 1$, $z_0 = 0$ and $z' < 0$, these conditions are reduced to

$$g\left(U_0^{||} + U_0^{\perp}\right) = u_1 \left(\frac{u_1}{g} D_1^{||} - D_1^{\perp}\right) + u_1 \exp(-u_1 z'), \tag{D7}$$

$$\left(U_0^{||} + U_0^{\perp}\right) = -\left(\frac{u_1}{g} D_1^{||} - D_1^{\perp}\right) + \exp(-u_1 z'), \tag{D8}$$

$$gU_0^{||} = -u_1 D_1^{||} + g \exp(+u_1 z') \tag{D9}$$

and

$$U_0^{||} = D_1^{||} + \frac{g}{u_1} \exp(+u_1 z'). \tag{D10}$$

Eqs (D7)–(D10), combined with eqs (D5) and (D6), yield the results described in eqs (44)–(47).